Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Reihe Ökonomie / Economics Series No. 39

Auctions - A Survey

René Böheim, Christine Zulehner

Auctions - A Survey

René Böheim, Christine Zulehner

Reihe Ökonomie / Economics Series No. 39

December 1996

René Böheim
ESRC
University of Essex
Colchester C04 3SQ
United Kingdom
Phone: ++44 (0) 1206-873901
Fax: ++44 (0) 1206-873151
e-mail: rene@essex.ac.uk

Christine Zulehner
Department of Finance
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna, Austria
Phone: ++43/1/599 91-150
Fax: ++43/1/599 91-163
e-mail: zulehner@ihssv.wsr.ac.at

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

The Institute for Advanced Studies in Vienna is an independent center of postgraduate training and research in the social sciences. The **Economics Series** presents research done at the Economics Department of the Institute for Advanced Studies. Department members, guests, visitors, and other researchers are invited to contribute and to submit manuscripts to the editors. All papers are subjected to an internal refereeing process.

Abstract

The importance of auction theory has gained increased recognition in the scientific community, the latest recognition being the award of the Nobel price to Vickrey and Mirrlees. Auction theory has been used in quite different fields, both theoretically and empirically. This paper connects recent developments in theoretical and empirical works, providing a survey about theoretical, empirical, and experimental results.

Keywords
Auction theory, information and uncertainty, asymmetric and private information.

JEL-Classifications
D44

Comments

We would like to thank Arno Riedl for his comments. All remaining faults are ours.

1 Introduction

Auctions are one of the oldest forms to determine transaction prices. One of the earliest reports of an auction is attributed to the Greek historian Herodotus. He described the sale of women to men willing to get married in Babylonia around the fifth century B.C.. Historical sources tell us about various auctions taking place in Greece, the Roman Empire, China, and Japan. Not only history is witness of this economic institution, but also nowadays auctions are used in a remarkable range of situations. There are auctions for livestock, flowers, antiques, artwork, stamps, wine, real estate, publishing rights, timber rights, used cars, contracts and land, and for equipment and supplies of bankrupt firms and farms. Auctions are of special interest to economists because they are explicit mechanisms, which describe how prices are formed.

The continuing popularity of auctions makes one wonder about the reasons why. One explanation is that auctions often yield outcomes that are efficient and stable. Or to say it more formally, in a static deterministic model, the set of perfect equilibrium trading outcomes obtained in an auction game (as the minimum bid is varied) coincides with the set of core allocations(Milgrom [43]).

A second explanation might be that a seller in a relatively weak bargaining position, consider the case where the seller is the owner of a nearly bankrupt firm, can do as well as a strong bargainer by conducting an auction (Milgrom [43]). However, she then can not use strategic policies like imposing a reserve price or charging entry fees. Even a seller in a strong bargaining position will decide to sell via auction, if it is optimal in relation to other exchange institutions. These three partly complementary explanations provide a cogent set of reasons for a seller to use an auction when selling an indivisible object.

The four most common auction forms are the first and second price sealed-bid, the English, and the Dutch auction. The description of all these kinds is given in Section 1.1.

Depending on what kind of good is to be sold we talk about private or common values auctions. The private value assumption is most nearly satisfied for nondurable goods. Because we can say that the consumption of such a good is a personal matter. In contrast, if we consider durable goods the private value assumption is not fulfilled anymore. There is the possibility of resale and therefore there is a market price. The difference between these two auction forms is described in more detail in Section 2.

Usually the seller and the bidders are assumed to be risk-neutral. Nevertheless there are papers dealing with risk-averse bidders. The same is true for symmetry. Symmetry or asymmetry among bidders means that we have to take into consideration whether the buyers draw their signals from a symmetric or an asymmetric probability distribution. The former implies that all bidders are homogeneous, whereas the latter allows heterogeneity among them.

The paper is organized as follows: In Section 3 we give a short overview about optimal auction design. In Section 4 a general auction model is introduced. Section 4.1 gives the equilibria for the four most common auction types. We also have a look at auctions with reserve prices and entry fees and what happens if we allow the bidders to be risk averse. Section 5 considers auctions with asymmetry, royalties, and collusion. In Section 6 the most prominent results of case studies concerning auctions used in "the real world" are compared, whereas Section 7 gives a summary of experimental outcomes. The preceding Section 8 deals with the econometrics of auctions. Results and an overview of used methods are given. The concluding Section 9 gives a summary and refers to those auction types, which have been omitted.

1.1 Description of Various Auction Forms

First and Second-Price Sealed Bid Auction The first-price auction is a sealed bid auction in which the buyer with the highest bid obtains the object and pays the amount she has bid. Whereas in the second-price auction the item still goes to the bidder with the highest bid, but she pays only the amount of the second highest bid. This arrangement does not necessarily mean a loss of revenue for the seller, as in this auction form the buyers will generally bid higher than in the first-price auction. The second-price auction is also known as "Vickrey" auction.

Dutch Auction The Dutch auction, also called descending auction, is conducted by an auctioneer who initially calls for a very high price and then continuously lowers the price until some bidder stops the auction and claims the good for that price. This kind of auction is frequently used in the agricultural sector.

English Auction There is more than one variant of the English auction. In some the bidders themselves are calling the bids and when nobody is willing to raise the bid anymore the auction ends. Another possibility is that the auctioneer calls the bids and the bidders indicate their assents by a slight gesture. Yet there is another form of the English auction. There the price is posted using an electronic display and is raised continuously. A bidder who is active at the current price presses a button. In the moment she releases the button she has withdrawn from the auction. This variant is in particular used in Japan. These are three quite different forms of the English auction with three quite different corresponding games.

2 Private vs. Common Value Auctions

Differences among the bidders' valuations of the auctioned object can have two possible reasons: differences in the bidders' tastes or the bidders have access to different information.

To be more explicit, suppose that each bidder knows exactly how highly she values the item to be sold. Further she knows nothing about the other bidders valuations, she perceives any other bidder's valuation as a draw from some probability distribution and she is aware that the other bidders regard her valuation to be drawn from some probability distribution as well. These probability distributions are common knowledge and the valuations of the bidders are statistically independent. In general this is called the *independent private value model.*

Now consider the case where the auctioned object has a value known to everyone. Namely the amount of this item on the market. However nobody knows the true value, but has some information about the item. And the bidders' perceived values are conditional on the unobserved value independent draws from some probability distribution. This is called the *common value model* or the *mineral rights model.*

The first situation applies, for example, to an auction of an antique, where the bidders buy for their own and will not resale it. Whereas in the latter situation an auction for an antique that is being bid for by dealers who intend to resell it is described.

The independent private value model and the common value model describe two extremes. In reality one might find auction situations lying between these two cases. A general model that allows for correlation among the bidders' valuations and that takes into consideration the two above described special cases, was developed by Milgrom and Weber [44]. This general model is explained in more detail in Section 4.

To go back to the example given above, an auction of an antique cannot be fully described in either of the two extreme cases. As the dealers may be guessing about the ultimative market value of the object, but they may differ in their selling abilities, so that the market value depends on which dealer wins the bidding. This arguing asks for a more general model.

2.1 The Independent Private Value Model

Much of the existing literature on auction theory deals with the independent private value paradigm in a risk neutral setting. In this section we now want to list the results and conclusions emerging from the independent private value model. As described above, in that model a single indivisible good is to be sold to one of n bidders. Any of the bidders knows the value of the item to herself, and nothing about the values of the other bidders. The values are then modeled to be independent draws from some continuous probability distribution. As the bidders are assumed to behave competitively, i.e. there are no collusions, the auction can be treated as a non-cooperative game.

There are several important results for this game (see among others Milgrom and Weber [44]):

Res.1 The Dutch and the first-price auctions are strategically equivalent.

Res.2 In the context of the private value model the English and second price auctions are equivalent as well, although in a somewhat weaker sense than the "strategically equivalence" of the Dutch and second-price auctions. That means, that in the latter case no assumptions about the values to the bidders of various outcomes is required. In particular, the bidder does not need to know the value of the item to herself.

Res.3 The outcome of the English and second-price auctions is Pareto optimal. In symmetric models the Dutch and first-price auctions also yield Pareto optimal allocations.

Res.4 The expected revenue generated for the seller by a given mechanism is precisely the expected value of the object to the second-highest evaluator.

Res.5 All four auctions, i.e. first-price, second-price, Dutch, and English [1], lead to identical expected revenues for the seller (Vickrey [56], Riley and Samuelson [53]). This is the so-called *revenue-equivalence* result.

Res.6 For many common sample distributions - including the normal, exponential and uniform distributions - the four standard auction forms with suitably chosen reserve prices or entry fees are optimal auctions (Myerson [45], Riley and Samuelson [53]).

Res.7 In case of either a risk-averse buyer or risk-averse seller the seller will strictly prefer the Dutch or first-price to the English or second-price auction.

2.2 The Common Value Model

In the common value auction we assume that the bidders make (conditionally) independent estimates of the common value of the item to be sold. The bidder making the largest estimate will make the highest bid. A consequence of this bidding strategy is that the winner will find that she had overestimated on average the value of the item she has won, even if all other bidders are making unbiased estimates as well. This

[1] In the remainder of the paper we denote these four kinds of auctions to be the four standard auction forms.

phenomenon is known as the *winner's curse*. It was first described by Capen, Clapp and Campbell [7], who claimed that this phenomenon is the reason for the low profits earned by oil companies on offshore tracts in the 1960's in the US.

In the case of first-price auctions the equilibrium of this model has been studied extensively. Those results dealing with the relations between information, prices, and bidder profits are one of the more interesting ones. For example, Milgrom [41] and Wilson [57] showed that – under certain regularity assumptions – the equilibrium price in a first-price auction is a consistent estimator of the true value, i.e., that although no bidder knows the true value of the item, the seller will receive that value as the sale price. In a common value auction the price can therefore be effective in aggregating private information. Furthermore, the bidder's expected profits depend more on the privacy than on the accuracy of the information about the common value of the good (Milgrom [42], Milgrom and Weber [44]).

3 Optimal Auctions

The theory of optimal auction design addresses the question which auction maximizes the expected revenue of the seller, given a single object to sell. The decision which kind of auction is the best is a problem of decision in the face of uncertainty. The seller does not know the value of the item to be sold to the bidders. Otherwise she would announce a non negotiable price at or just below the highest bidder's valuation. However, as the seller does not know the bidder's true valuations she is forced to choose among auction mechanism which are almost surely going to give her less than this perfect information optimum (Myerson [46]).

The tool to answer this question is the Revelation Principle. It shows that the seller can restrict herself to the class of direct and incentive compatible mechanisms. In a direct mechanism each bidder is simply asked to report her true valuation. Whereas in an incentive compatible mechanism the bidder finds it in her own interest to report her valuation honestly. The direct revelation game has one equilibrium that leads to the same allocation as the original equilibrium. But this equilibrium need not be unique (see also Fudenberg and Tirol [15]).

The optimal direct mechanism is found as the solution to a mathematical programming problem with two constraints: First, incentive-compatibility or self-selection constraints, which state that the bidders cannot gain by not truthfully reporting their valuations and second, individual-rationality or free-exit constraints, which guarantee that the bidders are not better off if they refuse to take part.

In most of the literature on optimal auction design the independent private value assumption is made. In this setup following results have been shown:

Res.1 The auction that maximizes the expected price has the following characteristics: (i) The seller optimally sets a reserve price and does not sell the item if all bidders' valuations are too low. (ii) Otherwise she sells to the bidder with the highest valuation (Myerson [45], Riley and Samuelson [53], Milgrom [43]).

Res.2 Any of the English, Dutch, first-price, and second-price auctions is the optimal selling mechanism provided it is supplemented by the optimally set reserve price (Myerson [45], Riley and Samuelson [53]).

Bulow and Roberts [6] have shown that the seller's problem in devising an optimal auction is virtually identical to the monopolist's problem in third-degree price discrimination.reinterpretation of the auction

The question whether a public auction or an optimally structured negotiation is more profitable to sell a company was answered by Bulow and Klemperer [5]. They found that under standard assumptions, like risk-neutrality, independence of signals, or increasing bid functions, the public auction is always preferable. The result holds both for the independent private value and the common value model.

Relaxing the Independence Assumption Myerson [45] has provided an optimal auction mechanism for an example with dependent bidders' values. The optimal auction includes side-bets, which are not possible in the independent case. In the general non-independent case we can expect side-bets more commonly. With carefully designed side-bets the seller can counterbalance the bidders' incentive to lie to buy the object at a lower price.

4 A General Symmetric Model

In Milgrom and Weber's paper [44] a general model for risk neutral bidders was developed. In the their model there is space for cases like the independent private value model and the common value model, as well as a range of intermediate models. In this section we will heavily draw on this paper.

Consider now an auction in which a single object is to be sold and in which n risk neutral bidders compete for the possession of that object. Each of these bidders has some information about the object.

Let $X = (X_1, \ldots, X_n)$ be a vector. The components of this vector are real-valued *signals* observed by the individual bidders. And let $Y_1, \ldots, Y_{n-1}$ denote the largest, …, smallest estimates from among $X_2, \ldots, X_n$. Let $S = (S_1, \ldots, S_m)$ be a vector of additional real-valued variables which influence the value of the object to the bidders. The seller might observe some of the components of S. Let $V_i = u_i(S, X)$ denote the actual value of the object to bidder i, and $f(s,x)$ denotes the joint probability density of the random elements of the model. Following assumptions are made:

Ass.1: $\exists$ function u on R^{m+n} such that $\forall i$, $u_i(S,X) = u(S, X_i, \{X_j\}_{j \neq i})$. That means, that all of the bidders' valuations depend on S in the same manner and that each bidder's valuation is a symmetric function of the other bidders' signals.

Ass.2: u is nonnegative, continuous, and nondecreasing in its variables.

Ass.3: For each i, $E[V_i] < \infty$.

Ass.4: The bidders' valuations are in monetary units and the bidders are risk neutral. Therefore, bidder i's payoff is $V_i - b$, if she receives the auctioned object and pays the amount b.

Ass.5: f is symmetric in its last n arguments.

Ass.6: The variables $S_1, \ldots, S_m, X_1, \ldots, X_n$ are affiliated. More precisely, let x and x' represent a pair of $(m+n)$ vectors, let $f(x)$ denote the joint probability density of the random variables x, and let $x \vee x'$ and $x \wedge x'$ denote the component-wise maximum and minimum of x and x', respectively. Then the variables are defined to be affiliated if, $\forall x, x'$,

$$f(x \vee x')f(x \wedge x') \geq f(x)f(x'). \qquad (1)$$

Roughly speaking, this condition means that large values for some of the variables make the other variables more likely to be large rather than small (Milgrom and Weber [44], p.1098). We refer to (1) as the "affiliation-inequality".

In this general setting both the independent private value paradigm and the common value paradigm can be treated. In the first case $m = 0$ and each $V_i = X_i$. Therefore the only random variables are $X_1, \ldots, X_n$. They are statistically independent and fulfill (1) with equality, i.e. independent variables are always affiliated.

In the second case $m = 1$ and each $V_i = S_1$. Let $g(x_i|s)$, $h(s)$ and $f(s,x) = h(s)g(x_1|s) \ldots g(x_n|s)$ denote the conditional density of any X_i given the common value S, the marginal density of S and the joint density of any X_i and S, respectively. Assuming that the density g fulfills the monotone likelihood ratio property [2] one gets that g also fulfills Equation (1). It can be shown that f also fulfills Equation (1) [3]. As a consequence the common value model defined in Section 2.2 meets the the formulation of the general model, if the density g has the monotone likelihood ratio property.

[2]Fore a definition see Section A.1.

[3]This can be shown by applying Theorem 1, which is listed in the Appendix (see Milgrom and Weber [44]).

As the auction is supposed to be symmetric we can focus on bidder 1 [4]. We can now rewrite bidder 1's value as follows:

$$V_1 = u(S_1, \ldots, S_m, X_1, Y_1, \ldots, Y_{n-1}). \tag{2}$$

The joint density of $S_1, \ldots, S_m, X_1, Y_1, \ldots, Y_{n-1}$ is

$$(n-1)! f(s_1, \ldots, s_m, x_1, y_1, \ldots, y_{n-1}) \mathbf{1}_{\{y_1 \geq \ldots \geq y_{n-1}\}},$$

where the last term is an indicator function. If the condition in parenthesis is met it is equal to one, otherwise it is equal to zero. It can be shown that the function $E[V_1|X_1 = x, Y_1 = y_1, \ldots, Y_{n-1} = y_{n-1}]$ is nondecreasing in x [5].

4.1 Results

We now list Milgrom and Weber's results concerning the optimal strategies in the standard auction forms. In this symmetric bidding environment they identify competitive behaviour with symmetric Nash equilibrium behaviour. Further we give a summary of results concerning revealing information, entry fees, reserve price, and risk aversion.

4.1.1 Second-Price Auction

A strategy for bidder i is a function mapping her value estimate x_i into a bid $b = b_i(x_i) \geq 0$. Supposing bidders $j \neq 1$ adopt strategy b_j then the highest bid among them will be $W := max_{j \neq 1} b_j(x_j)$. Bidder 1 will win the auction if her bid exceeds W, which will also be the price bidder 1 has to pay. The decision problem bidder 1 is facing now is to choose a bid b that maximizes the expected actual value minus the price conditional on her signal, ignoring the cases where her bid is not the highest. It can be shown that the equilibrium strategy in a second-price auction is to bid $b^*(x) = v(x, x)$ for every player. The function v is defined by $v(x, y) := E[V_1|X_1 = x, Y_1 = y]$. It can be shown that v is nondecreasing.

The expected selling price is the expected price paid when bidder 1 wins conditional on her winning the auction: $R = E[v(Y_1, Y_1)|\{X_1 > Y_1\}]$. Publicly revealing some credible information X_0 by the seller raises the expected selling price and therefore revenues. That means announcing information X_0 is better, on average, than revealing no information. Furthermore it can be shown that in a second-price auction, no other reporting policy, i.e. to censor information sometimes, yield a higher expected price than the policy of always reporting information X_0.

[4] From now we will retain this simplification. It allows us to identify the winning bidder with bidder 1, which will in particular be used in section 4.1.

[5] For this purpose Theorems 2-5 are engaged, which are all listed in the Appendix

4.1.2 English Auction

As in section 1.1 described there are several variants of the English auction. Milgrom and Weber developed a game model which corresponds most closely to the Japanese, so called press-button, auction: In the very beginning all bidders are active at a price of zero. The auctioneer raises the price and the bidders drop out one by one and can not become active anymore. Furthermore all other bidders know the price at which someone has quit the auction. A strategy for bidder i specifies whether, at any price level p, she will stay in the auction or not, as a function of her signal, the number of bidders having already quit, and the price levels at which they quit. Let k and $p_1 \leq \ldots \leq p_k$ denote the number of bidders who have quit and the levels at which they leave, respectively. Then bidder i's strategy can be described by a function $b_{ik}(x_i \mid p_1, \ldots, p_k)$ which specify the price at which bidder i will quit if, at that point, k other bidders have left at the prices $p_1, \ldots, p_k$. Naturally $b_{ik}(x_i \mid p_1, \ldots, p_k)$ is required to be at least p_k.

Milgrom and Weber showed that there exists a symmetric equilibrium point in this setup. The optimal bidding strategy $b^* = (b_0^*, \ldots, b_k^*)$ for any bidder is defined iteratively

$$\begin{aligned} b_0^*(x) &= E[V_1 \mid X_1 = x, Y_1 = x, \ldots, Y_{n-1} = x] \\ b_k^*(x \mid p_1, \ldots, p_k) &= E[V_1 \mid X_1 = x, Y_1 = x, \ldots, Y_{n-1} = x, \\ &\qquad b_{k-1}^*(Y_{n-k} \mid p_1, \ldots, p_{k-1}) = p_k, \ldots, \\ &\qquad b_0^*(Y_{n-1}) = p_1]. \end{aligned}$$

where $b_0^*(x)$ defines the optimal bid as long as nobody has left the auctions and $b_k^*(x \mid p_1, \ldots, p_k)$ is the equilibrium bid after k bidders have quit.

Further can be shown the English auction is not less than in the second-price auction. In effect, the English auction can be divided into two parts: First, the $n-2$ bidders with the lowest estimates reveal their signal publicly through their bidding behaviour. The last two players are then engaged in a second-price auction. Again, revealing information X_0 publicly raises revenues and no other reporting policy yields a higher expected price than the policy of always reporting X_0.

4.1.3 First-Price and Dutch Auction

Because of the strategic equivalence of the first-price and Dutch auction both auction forms can be treated equally. First, Milgrom and Weber derived necessary conditions for an n-tuple $(b^*, \ldots, b^*)$ to be an equilibrium point, when b^* is increasing and differentiable. Then the optimal strategy for any bidder in this setup is defined as follows

$$b^*(x) = v(x,x) - \int_{\underline{x}}^{\overline{x}} L(\alpha \mid x) dt(\alpha) \qquad \text{with}$$

$$t(x) = v(x,x) \qquad \text{and} \qquad L(\alpha \mid x) = exp(-\int_{\alpha}^{x} \frac{f_{Y_1}(s \mid s)}{F_{Y_1}(s \mid s)} ds).$$

where F_{Y_1} denotes the cumulative distribution of Y_1 corresponding to the density f_{Y_1}.

Additionally Milgrom and Weber proved that the expected selling price in the second-price auction is at least as large as in the first-price auction. And again, a policy of publicly revealing the seller's information can not lower, and may raise, the expected price in a first-price auction. As in the two other auction forms no reporting policy leads to a higher expected price than the policy of always reporting X_0.

4.1.4 Reserve Prices and Entry Fees

In Section 4.1.1- 4.1.3 any mention of the seller setting a reserve price or charging an entry fee has been omitted. Although in auctions these tools are commonly used and are thought of to raise the seller's revenue. Milgrom and Weber have shown how to adapt the optimal strategies in the case of a first-price auction. With a fixed reserve price the ordering of average prices does not alter. The English auction generates higher expected prices than the second-price auction, which in turn produces higher expected prices than the first-price auction does. However, given any reserve price for one of the standard auction forms a policy of announcing information X_0 and setting the corresponding reserve price raises expected revenues. A further result shows under some additional conditions, that it pays to set high entry fees and low reserve prices, rather than the reverse.

4.1.5 Risk Aversion

Risk aversion raises the expected selling price. In models combining risk aversion and affiliation it is generally not possible to rank the first- and second-price auction by their average prices. This is in contrast to the case of independent private values with risk aversion where the first-price auction yield higher prices than the second-price auction.

Generally it is not true that partially resolving uncertainty reduces the risk-premium. The class of utility functions for which any partially resolving uncertainty tends to reduce the risk premium is a very narrow one. It is the class of increasing utility functions with constant absolute risk aversion.

If one assumes that bidders are risk averse with constant absolute risk aversion, then in the second-price and the English auction revealing public information raises the expected price and among all possible reporting policies for the seller full reporting leads to the highest expected price. Additionally, the expected price in the English auction is at least as large as in the second-price auction.

4.1.6 Summary

When bidders' valuations are affiliated, the English auction yields a higher expected revenue than the first-price, the second-price, or the Dutch auction. Additionally it is true that the second-price auction leads to higher expected revenue than the first-price auction, which yields the same revenue as the Dutch auction.

In the standard auction forms the seller can raise her expected revenue by having a reporting policy of revealing any information she has about the item's true value. If one bidder's information is available to another bidder, her expected surplus is zero (see Milgrom[42], Milgrom and Weber [44]). This result means that privacy of information is of more importance than precision of the information.

5 Further Topics

We now consider the assumptions of the independent private value model. These are independent private values, bidders' risk neutrality, and symmetry, and that payment is a function of bids alone. Relaxing the first assumption leads to the common value or general model (see Section 2.2 and 4). Results concerning risk aversion are given in Section 4.1.5. The next two Sections 5.1 and 5.2 are dealing with relaxing one of the last two assumptions, whereby the first two are recovered again. The last Section 5.3 deals with collusion among bidders.

5.1 Asymmetric Bidders

Following McAfee and McMillan [35] we now assume that the bidders fall into separate groups. Therefore we do not have one distribution of private values anymore, but two different ones. Bidders of every type draw their valuations still independently from their specific probability distribution.

In this setting the English auction operates much as in the private value model. The bidder with the highest valuation wins and therefore the outcome is efficient. As the first-price auction yields a different price as the English auction does, revenue equivalence breaks down. Vickrey [56] among others has constructed examples in which the price in the English auction can be on average higher or lower than the price in the first-price auction.

In the special case that the two distributions are only distinguishable by their mean, i.e. the shape of the distribution is the same, only the means differ, the class of bidders with the lower average valuation are favoured in the optimal auction (McAfee and McMillan [35]). The benefit from such a policy is that the bidders with the higher average valuation are forced to bid higher than they otherwise would. Thus the price will drive up on average.

5.2 Royalties

So far payment was dependent only on bids. Consider now the auction of publishing rights for books with payment to the author depending on the bid and, via royalty, on the book's ultimative sales. It is in the seller's interest to condition the bidders' payments on any information about the winner's valuation (McAfee and McMillan [35]). Ex post the seller observes an estimate of the winning bidder's true valuation. In this setup the payment to the seller by the winning bidder is modeled as a linear function of the bid and the royalty rate times the estimate of the winning bidder's true valuation. There are three possibilities for bidding mechanism for this linear relationship. First, the seller can set the royalty rate and call for bids. Second, she can set the fixed payment and call for bids on the royalty rate, and third she can call on both.

Considering the first type of bidding mechanism the following results can be shown: If the distribution of the observed estimate of the winning bidder's true valuation is exogenous, the seller's expected revenue is an increasing function of the royalty rate (McAfee and McMillan [33]). This suggests a royalty rate of hundred percent. An increasing royalty rate also serves to increase the bidding competition and raises the bids. However, an increase in the royalty rate reduces the return to the winning bidder on her own actions after the auction. This in turn tends to lower the seller's expected revenue. With moral hazard the optimal royalty is less than hundred percent and the optimal contract is linear in the observed estimate of the winning bidder's valuation but nonlinear in the winning bidder's bid (McAfee and McMillan [35]). Considering risk-averse bidders one gets that the higher bidders' risk aversion compared to the seller's, the higher the optimal royalty rate should be (McAfee and McMillan [33], Samuelson [54]).

The second type of bidding mechanism has been investigated by Riley [52]. He showed that under certain assumptions the expected payment by the winning bidder is strictly greater under royalty bidding than under pure fee bidding and that the expected revenue from a high bid auction is a strictly increasing function of the royalty rate.

Samuelson [54] has taken the more general case, where the bidder submits a bid represented by an ordered pair, one bid for the royalty rate and one bid for a fixed payment. The seller "orders" the bids according to a preannounced selection function. Under these assumptions the seller seeks a selection function which maximizes her expected revenue.

5.3 Collusion

So far it was always assumed that the bidders act non cooperatively, i.e. they do not coordinate their bids. However, in reality this assumption is not always met. Collusion between bidders in form of cartels or bidding rings are built to prevent too high prices.

In English auctions a common method in practice is for one member arbitrarily to be assigned to bid for the item without competition from his fellow cartel members. Afterwards, the item is re-auctioned among the cartel members (McAfee and McMillan [33]). Milgrom [43] examined that auction forms differ in their degrees of susceptibility to collusion. According to Mead's hypothesis [40] he pointed out that in an ascending auction collusion is easier to support than in sealed-bid auctions. This would explain why a seller might choose a sealed-bid auction. This holds despite the fact that the ascending auction is theoretically superior when bidders behave competitively. Instead of changing the auction form the seller can also use a reserve price. Under the private value assumptions it can be shown that the optimal anti-cartel reserve price can be calculated from a relation, where the seller's valuation is a function of the reserve price, and the number of bidders (McAfee and McMillan[33]). This formula also implies that the anti-cartel reserve price increases with the number of cartel members and that the anti-cartel reserve price is higher than the optimal reserve price without collusion.

6 The Real World

Empirical work on auctions have mostly been asking questions like "Does the bidder with the highest valuation get the object?" or, "Does the seller get all profit?", etc. That is to say, questions concerned with *efficiency* and *optimality* of the auction. Key parameters to these questions are the reserve prices and the mechanism designs.

In this section we will compare different markets where auctions have been used to sell the object. We will consider here only the most prominent results, since the current volume of EconLit (WinSPIRS) produces 162 hits for the search string "auction* AND empiri*".

6.1 Oil and Gas Lease Auctions in the U.S.A.

The most prominent authors concerning oil lease auctions are Hendricks and Porter. In their AER 1988 paper [18] they analyse drainage lease auctions on the Outer Continental Shelf (Louisiana and Texas) from 1959 to 1969. The reason for using this type of auctions is that the data set is very rich, i.e. it is possible to identify the buyers, the bidders, and, ex post, the profits of the oil fields. For each tract that receives at least one bid following information is obtainable from the U. S. Department of the Interior: date of sale, location and acreage, the identity of all bidders and the amount they bid, the identity of participants in joint bids, acceptance of the bid, number and date of drilled wells, monthly production through 1991 of oil, condensate, gas, and other substances.

The objects were classified in two categories, drainage and wildcat leases. A drainage sale consists of simultaneous auctioning of tracts which neighbour tracts on which deposits have been discovered. Wildcat tracts are in areas where no previous drilling has occurred. Bidding, drilling behaviour, and profits differ significantly on these two types of tracts.

The main difference between wildcat and drainage auctions is the distribution of information. For wildcat leases information is essentially symmetric, since no company is allowed to engage in explorative drilling. For drainage tracts the neighbouring firm has the possibility to obtain seismic information. Non-neighbouring firms have the possibility to obtain information from private seismic surveys and from observation of production by their competitors. Neighbouring firms tend to be better informed than the non-neighbouring rivals. This would give them an advantage and could result in the winner's curse for the actual winner of a lease.

The finding is that the data strongly support this hypothesis. The decisions of the neighbouring firms are significantly better predictors. The predictions of the Bayesian Nash equilibrium model of bidding in first-price, sealed bid auctions with asymmetric information are consistent with the data. Non-neighbouring firms earned approximately zero profits whereas neighbouring firms were able to gain substantially higher returns.

Hendricks and Porter [20] focus on the role of joint bidders in OCS auctions. They again take data for the period 1954 till 1979 for U.S. land off Louisiana and Texas. The hypothesis that joint ventures enhance competition by allowing smaller firms (the "fringe") entry to the market is investigated. The participation of the fringe happened predominantly via joint bidding, but usually with a big firm, not together with others from the fringe. However, in this sample the most profitable bids came from large companies bidding either solo or with each other. The participation of the fringe in joint ventures seems to be motivated mostly by capital constraints. This interpretation is consistent with several aspects, namely the size of the venture, differences of profits, etc. The discussion of this issue is taken up again in Porter [51], see also below.

In their European Economic Review article Hendricks and Porter [21] analyse oil lease auctions off the coasts of Texas and Louisiana, U.S.A., by the U.S. federal government. The period investigated is 1954 to 1979, using first-price sealed bid auctions as the theoretical model.

The objects were classified in three categories, wildcat, development, and drainage leases. Drainage and development leases are adjacent to existing areas where oil has been found (after 1979 this distinction was no longer used), whereas wildcat leases are in regions where no previous drilling has been undertaken. In the case of drainage and development leases, owners of neighbouring tracts have data from their own drilling. For the wildcat leases potential buyers are allowed to gather seismic information but no on-site drilling is permitted. Development leases were mostly re-offers of previously sold tracts with relinquished leases, or of leases where the bids were rejected as too low.

The bid is a dollar amount which the firm agrees to pay to the government if it is awarded the lease. There is also a fixed fraction, known as royalty, of any production revenues that the firm has to pay to the government (in this case mostly 1/6).

To account for the role of information asymmetry the authors distinguished between neighbouring and non-neighbouring firms. A regression of ex post profits on participation and bids of the firms verified the informational advantage of the neighbouring company.

The findings of this paper are that information plays a crucial role, neighbouring firms earning significantly more than non-neighbouring. Furthermore, the returns for non-neighbouring firms were lower if neighbouring firms did not bid. These results are consistent with the theoretical predictions.

As in the above paper, Porter [51] analyses the sale of oil and gas tracts off Texas and Louisiana. The oil and gas production in this area accounts for about 12 percent of U.S. oil production (25 percent of gas production). The data set runs from 1954 to 1990 (till 1979 for wildcat sales). The article addresses both symmetric (wildcat leases) and asymmetric (drainage leases) situation, the role of the government, whether and how bidding consortia acted, and the incidence and timing of exploratory drilling after a tract was acquired.

In the wildcat lease auctions the presence of the winner's curse might be shown by the data. A substantial dispersion in the number of submitted bids is reported. Out of 2.255 purchased tracts on a fraction of 78 percent exploratory drilling was undertaken. Conditional on being explored, tracts receiving more bids are more likely to be productive and to have larger deposits if productive.

An interesting question is the behaviour once a site has been bought. Buying a tract does not force the buyer to engage in exploratory drilling, however, if no drilling is undertaken the lease is relinquished and reverts to the government. There is a clear deadline effect as the five year deadline approaches. This is consistent with a war of attrition if the firm prefers its neighbour to drill first. Relative to the optimal coordinated plan (where sequential search would be carried out) the non cooperative equilibrium is inefficient (delay of drilling).

The rejection policy of the government is studied. The government rejected bids more often if there were few bidders. It seemed that the government's rejection policy was to discourage low bids. Porter noted that the long period between rejection and reoffering may be inconsistent with revenue maximization.

Porter [20] looked at the behaviour of joint bidders for the period 1954–79, computing a ex post value for the tracts. The most profitable bids have been made from large firms bidding either solo or jointly with each other. The average ex post value was lower than that of tracts bought by consortia of large and small firms, but the lower purchase prices more than offset the difference. The participation of small firms was predominantly together with large firms for high-price tracts. This behaviour can be explained by capital constraints, however it cannot be the explanation of why large firms bid together. Here the motivation could be to reduce competition and lower the prices.

6.2 Auctions of Contracts

Paarsch [48] uses auctions of tree planting contracts in British Columbia, Canada, from 1985 to 1988, to find whether this auctions fall within the private or common value paradigm framework. The contracts are awarded to the lowest bid, which is typically the cost per tree planted, in a sealed bid first-price auction. Differences in bidding behaviour can be a result of two different (and opposing) paradigms. In the first case, the cost of planting a tree is unknown to all, but the same to all bidders. If this is true, then a sealed-bid framework offers no additional help in finding out the most efficient since all are equally efficient.

If, on the other hand, there are differences in the planting costs then the use of the sealed-bid framework will induce the bidder with the lowest cost to submit the lowest bid.

Which paradigm applies will be reflected in the bidders' bidding functions and in the winning bids. For common value auctions, the bid functions will increase in the number of bidders, n, where in auctions within the private value setting the bids will decrease monotonically. The expected value of the winning bid will have no relationship with the number of bidders in a common value setting, in private value auctions it will decrease in n.

Sadly, the distribution of the winning bids do not provide enough information to discriminate between the models. Neither the shape nor the mean of the data allows discrimination. Employing several empirical specifications (proportional bids, cost additive bid functions, nonlinear functions of cost) all versions of the private value paradigm are rejected and there is consistency between the data and the common value paradigm.

Pesendorfer [50] investigates collusion behaviour in milk contract auctions in Florida and Texas, U.S.A., from 1980 to 1991. Here a first-price sealed bid mechanism is employed, too. The lowest bid is accepted, all bidders have to sign a non-collusion statement, using this framework some 200 million \$in school milk contracts were auctioned between 1980 and 1989, around 200 contracts per year. Between 1988 and 1993 45 companies and 26 persons have been convicted for bid-rigging. There are several aspects in this specialised market that may encourage collusion. Firms have to agree only on a price since quality and quantity of milk contracts are prespecified. On top of that there are many contracts, so that profits can be divided amongst partners. There is no coordination between different districts, the contracts are awarded only on the bids submitted to the specific Board of Education. Colluders can detect defiants easily (announcement of bids) and react within the same year. The demand elasticity is very low and the market highly concentrated (8 firms won 90 percent of all contracts).

The author finds that both markets are consistent with game theoretic models. In the case of Florida, a cartel with sidepayments, in Texas, a cartel without side-payments are consistent with the data.

In a very influential[6] paper Thiel [55] investigated the highway construction auctions. In this setup contracts are awarded to qualified firms entering the lowest bid. Usually the highway agency does not reveal its estimate of the value of the contract[7]. The author uses a GLS regression under the hypothesis that the sampling distribution is normal to identify the bidding behaviour. There is consistency in the sense that bidders seem to be aware of the winner's curse and modify their bids accordingly. The drawback of the model, highly critised, is the specific econometric specifaction right down to the functional forms.

6.3 Radio

McMillan [39] investigated the mechanisms used in the selling of radio frequences in the USA.

The author reports the difficulties New Zealand faced when TV licenses were auctioned: Second-price auctions were used, but no reserve price was imposed. This led to a massive loss for the government (after massive critisism now first-price sealed bid auctions are used).

The Federal Communications Commission (FCC) chose to use a very complicated mechanism, a simultaneuous open multiple-round auction. An open auction was chosen since it reduces the winner's curse. The FCC chose to run multiple rounds of sealed bids and announced the bids after each round. This simulates an open outcry auction giving extra control to the government. With this setup the government has control over collusion, since bidders do not know with whom they are competing. Reserve prices were used only for licenses that attracted three or fewer bids. Bidders who withdraw a winning bid, or, in case of minorities, who had privileged access to frequencies, resold their right, had to pay a penalty for doing so. The FCC did not use royalties, although theory shows that royalties benefit the seller.

[6]This paper led to a debate, some of which was reprinted in the AER 1991, cf. Levin and Smith [32], Kagel and Levin [27], and Hansen and Lott [17].

[7]There are agencies that do tell the estimated value. But this causes drastic change in the bidding behaviour and is not dealt with in the article.

6.4 Other Markets

Laffont, Ossard and Vuong [30] used data from French eggplant auctions to test their model proposed in the article (the method used will be presented in Section 8). The obtained results are consistent with the theory, i.e. higher quality receives higher prices. The relative importance of local produce is somewhat surprising but not inconsistent.

McAfee and Vincent [36] and Ashenfelter [1] investigated wine auctions and noted something that became known as the "price-decline anomaly".

Ashenfelter and Genesove [2] investigated auctions of condominiums. The mechanism that is used to sell condominiums is a "pooled auction", in which all units are combined and the highest bidder has the right to choose among the units. This procedure is repeated untill all the products are sold, resp. no one bids any longer. Prices usually decline since the early bidder pay a premium for the selection right. Three important findings were reported: First, the prices did decline in the progress of the auction. Second, the prices of the items showed little or no relationship to the order of selling. Third, a large price discount was seen in items sold at the beginning of the auction, which is evidence that the prices did not reflect higher quality. The buyers fell pray to the winner's curse.

McAfee and Vincent [37] tested for the optimal participation in these auctions. They have found that the reserve price should be raised, thereby lowering the participation rate and increasing government's expected revenue. The reserve price, according to their model (see Section 8), should be 40 times higher than it was.

7 Experimental Auctions

Hendricks and Paarsch [19] see the role for auctions in experiments to test the behavioural predictions of game theoretic analysis. If the valuations and the underlying probability distribution is know to the experimenter, than a comparison of the equilibrium strategy and agents' behaviour can tell something about the relevance of the theory.

Another important issue is that each year many goods are actually auctioned, from art to wine. Auctions constitute one of the simplest way of price determination and are easily carried out.

One predominant question in auction theory is the *winner's curse.* This question was first posed in the seminal work of Capen, Clapp, and Campbell [7], who noted that in case of oil fields the winning bid usually received unexpectedly low returns.

In experimental settings auctions have been used to analyse a wide field of issues, from bargaining models to political stock markets (double-sided auctions).

Kagel [24] gives a very detailed survey of results in experimental auctions. He himself draws on the work of Wilson [58] and McAfee and McMillan [34]. For the sake of completeness and comparability with Section 6 we will reproduce here the most important findings, listed also in Table 1.

Table 1: Questions Addressed with Auctions in Experiments.

Question	Auction type	Results	Reference
Winner's curse	Private value		
	· First-price sealed bid	Persistent deviation from RNNE[a]	Overview in Davis and Holt [11]
	· · with asymmetric information	-"-	
	· Second-price sealed bid	-"-	
	· English auctions	-"-	
	· Third-price auction	-"-	Kagel and Levin [29]
	Common value	Persistent deviation from RNNE	Overview in Wilson [58]
	· first-price	-"-	Lind and Plott [31]
	· English	-"-	Kagel and Levin [28]
	· asymmetry	-"-	Ball [3]
			Cifuentes and Sunder [9]
	· blind bid	N	Forsythe, Isaac, and Palfrey [14]
	· lemons	N	
	· learning	N	Cifuentes and Sunder [9]
		Y	Garvin [16]
Loser's curse	Common value	N	Holt and Sherman [22]
Strategic equivalence	First-price and Dutch auctions	N (first > Dutch)	Coppinger, Smith, and Titus [10]
	Second-price and English auctions	N (second > English)	Kagel. Harstad, and Levin [25]
			Kagel [29]
Affiliated private values	first-price	Y	Kagel, Harstad, and Levin [25]
Learning	first-price	+/-	Kagel [24]
Risk aversion	Several	+/-	Kagel [24][b]
Collusion	Common value	N	Lind and Plott [31]
		+/-	Isaac and Walker [23]
		Y	Kagel [24]

This table was compiled following Kagel [24]. Y or N in the result column indicates whether the results were according to theoretical predictions, +/- stands for controversial results.

[a] RNNE stands for risk-neutral Nash equilibrium.

[b] gives a detailed overview of ongoing research in this field.

The questions addressed deal with the most important questions theory is faced within the field analysis. These questions are – amongst others – the winner's curse, the strategic equivalence of certain types of auctions, whether learning does alter the bidding behaviour, and, if collusive behaviour can be studied.

7.1 Results

In field data the question of whether the winner's curse is present is not easily answered, the setting is usually very complex, in some cases (oil fields) the value is not known for years after the bidder bought the drilling rights.

In a seminal paper Bazerman and Samuelson [4] conducted an experiment mainly to find out if the winner's curse is present in a laboratory situation. They observe a clear winner's curse where the average winning bid was some 25 per cent over the value of the object (10 $ as opposed to 8 $).

In this experiment the main results could have come from other factors as well. The bidders were untrained and there were no repeated experiments. Kagel and Levin [26] designed an experiment to deal with these issues. The main result was that the winning bids were higher than the Nash equilibrium bids, but declined with participants' experience.

In the laboratory the real value of the object is known to the experimenter, this is not possible in the real world, where sometimes the real value of the object is known only to the victims of the winner's curse.

But clearly, all experiments reported the presence of the winner's curse, even the very unrealistic setup of Kagel, Harstad, and Levin [25] of the third price auction generates overbidding. So, this seems still to be a very important feature one has to bear in mind when using auctions. Several countries have already introduced guidelines against overbidding in selling of public contracts, see e.g. Finsinger [13].

Other results from experiments are not as easily interpreted than the former ones. The winner's curse being the most important researched topic in this context, other questions have only recently been investigated. And, furthermore, some of the results obtained do not allow a straightforward conclusion.

What can be said, however, is that there is a rich field of scientific research and that it is still very important to consider other influences to the outcome of auctions – like repeated participation or risk aversion.

8 The Econometrics of Auctions

Many recent papers in auction literature are concerned with the possibility of formulating and testing hypotheses. Hendricks and Paarsch [19] give an introduction to structural and non-structural models. Both draw on previous work (e.g. Hendricks and Porter [20] or Paarsch [48]) to distinguish between this two approaches.

One aim of recent work in auction theory was to identify the probability law behind the valuations of potential bidders. This is essential if one wants to implement an optimal auction mechanism. Bayesian Nash Equilibrium behaviour imposes restrictions upon the relationships amongst bidders (e.g. between informed and uninformed bidders) which do not depend on the functional form of the probability law for the valuations.

If one assumes that there is no unobserved heterogeneity the actual knowledge of this probability law is unnecessary to test these restrictions (therefore "non-structural approach").

In contrast to the former approach, among others Paarsch [47], [48] has proposed to use the entire structure of the auction to derive the data generating process. There is admittance of heterogeneity, observed and unobserved, in this approach (the "structural approach"). The complexity of the equilibrium bid functions are the main difficulty this approach faces.

8.1 The Non-Structural Approach

The sale of drainage leases as described in Section 6 was described as following a common value auction with a first-price sealed-bid mechanism.

Hendricks and Porter [20] imposed *affiliation* on the joint distribution of values, signals, and reserve price.

Following this approach Hendricks and Porter [21] derived three restrictions governing the behaviour of informed and uninformed bidders in the OCS auctions. First, a lower participation rate of non-neighbouring firms; second, few non-neighbouring bids below some de facto reserve price; third, distributional equivalence above some level (where bids are almost never rejected, a de facto acceptance price). They report tests on these restrictions and the empirical distributions of bids produced "remarkable strong support" for the theory. However, there are two caveats: Firstly, the distribution of data was truncated, but that can be accommodated by imposing a condition on the first order stochastic dominance. Secondly, the assumption of independent distribution might not be fulfilled and for this reason the asymptotic distribution of the test statistic is almost surely not standard (and up to some calculational difficulties).

Laffont, Ossard, and Vuong [30] propose a structural estimation method for the empirical study of first-price, sealed bid and descending auctions. This method is based on simulations relying on nonlinear least squares objective functions. The authors adopt the private value paradigm where each bidder is supposed to have a different private value drawn independently from a probability distribution and use the structure of the winning bid prediction as an estimate for the underlying structure.

McAfee and Vincent [37] use data from Hendricks and Porter [20] for OCS auctions. They extended the generalized mineral-rights model [8] to allow for stochastic endogenous participation.

In this model we have nature as a player that determines the number of possible bidders, n, with some probability q_n. Out of these a subset of the potential bidders is drawn at random. The true value is v and the bidders receive some signal, x_i, which is independently distributed. A theorem is derived that provides a distribution-free test to test whether participation is optimal. The auction is attracting inefficiently few bidders whenever net revenues exceed the values of tracts that attracted two or more bids.

8.2 The Structural Approach

8.2.1 English Auctions

Under the private value paradigm the setup of the English auction is the simplest to estimate the joint distribution function on the n valuations. This is because of two important assumptions: First, the independence of valuations, and, second, the failure to distinguish between buyers. This means, that the marginal distributions of the joint distributions are identical (n-times). If the task is to estimate F, the cumulative distribution function, this can be accomplished by recovering the entire joint distribution for the buyers' valuations. A parametric formulation can now be derived (see Hendricks and Porter [21] for a formal derivation):

$$F_t(v) = F(v; \theta, Z_t),$$

where Z_t is vector of characteristics which are observable, t indicates one auction out of T auctions, and θ is the unknown value. From this a likelihood-function can be calculated and tested, given the results of an experiment.

Hendricks and Porter [21] reports results from Paarsch [47] who used this methods in the British-Columbia timber auctions.

8.2.2 First-Price Sealed Bid Auctions

The winning bid in a first-price sealed bid auction, under independent private values, is a function of the set $\{v_i\}_{i=1}^{n}$. As in the previous section, $F_t(v) = F(v; \theta, Z_t)$ represents

[8] The generalized mineral rights model is the same as the common value model without the (conditional) independence assumption (Milgrom and Weber [44]).

a parametric specification for each auction t, where $t = 1, \ldots, T$. The difficulty in this framework is the fact that the upper bound of the support of the winning bid depends on the parameters of interest — this violates the asymptotic consistency of the maximum-likelihood estimator.

Paarsch [47] has used this framework for the aforementioned timber-sales. The results showed variation in F depending on the use of English auctions or first-price sealed bid auctions. The bidding behaviour differed in that the winning bid was more often the reserve price with English auctions than with first-price sealed bid auctions. Moreover, the amount of rent government accrued was negative.

8.2.3 Simulated Non-Linear Least Squares

The drawback of the last estimation method is the computational complexity. In general the involved functions will not have an analytic solution and have to be solved numerically.

The proposed alternative of Laffont, Ossard, and Vuong [30] is related to the method of simulated moments (McFadden [38], and Pakes and Pollard [49]). The simulated non-linear least squares estimator can be derived by minimizing the following objective function:

$$
\begin{aligned}
Q(\theta) \quad = \quad & \sum_{t=1}^{T} [(w_t - \bar{m}(\theta, Z_t, N_t))^2 - \\
& \frac{1}{J(J-1)} \sum_{j=1}^{J} (m_j(\theta, Z_t, N_t) - \bar{m}(\theta, Z_t, N_t))^2],
\end{aligned}
$$

where θ is the parameter to be estimated from finite (fixed) J simulations. $\bar{m}(\cdot)$ is an estimator for $\mu(\cdot)$, which is an estimate for $E[w_t]$. Asymptotically the estimation error generated by $\bar{m}$ is eliminated by the estimate for the sample variance, the second term in brackets. They have shown that the asymptotic distribution is normal.

8.2.4 Non-Parametric Estimation

The main criticism of the maximum likelihood method or the simulated non-linear least squares is that an explicit assumption concerning the density functions, $f(v)$, is needed.

The non-parametric approach, however, has to use all of the bids, not just the winning bid, which the above methods used. The assumptions that are made is that each potential buyer is bidding optimally against the opponent's bidding strategy, potential buyers are using the same strategy, and this strategy is increasing in v.

The procedure consists of two steps. The first step involves the estimation of the conditional density, $g_S(s|Z)$, in a non-parametric way. The estimation of the "hazard rate",

$\frac{g_S(s|Z)}{G_S(s|Z)}$, is then used to estimate the bidding-strategy and the conditional density of valuations. This provides a test of the behavioural hypothesis, which is carried out for timber sales (Elyakime, Laffont, Loisel, and Vuong [12]). The auction is interesting in that the seller's reserve price is not announced until the bids have been submitted. Theoretically, from the point of the buyers, the seller is another buyer, but with a different payoff function. This implies a different bidding strategy and a lot of trouble for the researcher.

9 Conclusion

In this paper we have given an overview on auctions with an indivisible single object to be sold to one of several buyers by one seller. Addressing the theoretical conditions for which this setup can be analysed as a non cooperative game we have listed the main results.

With these findings markets for quite different objects have been investigated. The problem is that the fundamental parameters of the game are neither known to the researcher, nor to the buyers themselves.

To pin down strategic behaviour two possible approaches are considered: First, experimental economics, second, econometric analysis. From experiments scientists can conclude from results to theoretical shortcomings, although – since the laboratory does not reflect the complexity of "real markets" – in a limited way. The econometric approach is still in its beginning, using quite distinct ways of modeling estimators.

One striking feature with auction theory is that theory *determines* the market setup, for instance designing the rules for radio frequencies auctions, whereas on the other hand effort is being made to analyse the *outcome* of market behaviour.

The models we have considered in this survey are the standard models. Some cases, which have been treated in the literature we have omitted, e.g. multiple and double-sided auctions. For other more complicated cases no results have been obtained yet.

We tried to show that auction theory can deal with complex strategies and might be a tool much needed in future research.

References

[1] Ashenfelter, O. (1989): "How Auctions Work for Wine and Art," *Journal of Economic Perspectives*, 3, 23-26.

[2] Ashenfelter, O. and D. Genesove (1992): Testing for Price Anomalies in Realestate Auctions," *American Economic Review (Papers and Proceedings)*, 82(2), 501-5.

[3] Ball, S.B. (1991): "Experimental Evidence on the Winner's Curse in Negotiations," *Diss. Northwestern University*, (quoted in Kagel [24]).

[4] Bazerman, M.H. and W.F. Samuelson (1983): "I Won the Auction but Don't Want the Prize," *Journal of Conflicts Resolutions*, 27, 618-34.

[5] Bulow, J. and P. Klemperer (1994): "Auctions vs. Negotiations," NBER Working Paper 4608.

[6] Bulow, J. and J. Roberts (1989): "The Simple Economics of Optimal Auctions," *Journal of Political Economy*, 97, 1060-1090.

[7] Capen, E.C., R.V. Clapp, and W.M. Campbell (1971): "Competetive Bidding in High-Risk Situations," *Journal of Petroleum Technology*, 23, 641-653.

[8] Cassady, R. (1967): *Auctions and Auctioneering*, Berkeley: University of California Press (quoted in McAfee and McMillan [33]).

[9] Cifuentes, L.A. and S. Sunder (1991): "Some Further Evidence on the Winner's Curse," *Mimeo. Caranegie Mellon University.*

[10] Coppinger, V.M., V.L. Smith, and J.A. Titus (1980): "Incentives and Behaviour in English, Dutch, and Sealed-Bid Auctions," *Economic Inquiry*, 43, 517-20.

[11] Davis, D.D. and C.A. Holt (1993): *Experimental Economics*, Princeton University Press.

[12] Elyakime, B., J.J. Laffont, H. Ossard and Q. Vuong (1994): "First-Price Sealed-Bid Auctions with Secret Reservation Prices," *Annales d'Econmie et de Statistique*, 34, 115-141.

[13] Finsinger, J. (1987): "The Consequences of Non-Competitive and Protectionist Government Purchasing Behaviour," *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 123(2), 147-73.

[14] Forsythe, R., R.M. Isaac, and T.R. Palfrey (1989): "Theories and Tests of "Blind Bidding" in Sealed Bid Auctions. *Rand Journal of Economics*, 20, 214-38.

[15] Fudenberg, D. and J. Tirole (1991): *Game Theory*, MIT Press, Cambridge Massachusetts.

[16] Garvin, S.(1994): Learning in Common Value Auctions: Some Initial Observations," *Journal of Economic Behaviour and Organisation*, 25(3), 351-72.

[17] Hansen, R.G. and J.R. Lott (1991): "The Winner's Curse and Public Information in Common-Value Auctions: Comment. *American Economic Review*, 81(1), 347-61. Theory," *American Economic Review, AEA Papers and Proceedings*, 75, 862-865.

[18] Hendricks, K. and R.H. Porter (1988): "An Empirical Study of an Auction with Asymmetric Information," *American Economic Review*, 78, 865-883.

[19] Hendricks, K. and H.J. Paarsch (1993): "A Survey of Recent Empirical Work Concerning Auctions," Working Paper, Department of Economics, University of Western Ontario.

[20] Hendricks, K. and R.H. Porter (1992): "Joint Bidding in Federal OSC Auctions," *American Economic Review (Papers and Proceedings)*, 82(2), 506-11.

[21] Hendricks, K. and R.H. Porter (1993): "Bidding Behaviour in OSC Drainage Auctions," *European Economic Review*, 37, 320-8.

[22] Holt, C.A. and R. Sherman (1994): "The Loser's Curse and Bidder's Bias," *American Economic Review*, 84, 642-52.

[23] Isaac, R.M. and J.M. Walker (1985): "Information and Conspiracy in Sealed-Bid Auctions," *Journal of Economic Behaviour and Organisation*, 6, 139-59.

[24] Kagel, J.H. (1995): "Auctions: A Survey of Experimental Research," in *The Handbook of Experimental Economics*, edited by J.H. Kagel and A.E. Roth, Princton University Press.

[25] Kagel, J.H., R.M. Harstad, and D. Levin (1987): "Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study. *Econometrica*, 55, 1275-1304.

[26] Kagel, J. and D. Levin (1986): "The Winner's Curse and Public Information in Common Value Auctions," *American Economic Review*, 76, 894-920.

[27] Kagel, J. and D. Levin (1991): "The Winner's Curse and Public Information in Common Value Auctions: A Reply," *American Economic Review*, 81, 362-9.

[28] Kagel, J. and D. Levin (1992): "Revenue Raising and Information Processing in English Common-Value Auctions," *Mimeo. University of Pittsburgh*, (quoted in: Kagel [24].

[29] Kagel, J. and D. Levin (1993): "Independent Private Value Auctions: Bidder Behaviour in First-, Second-, and Third-Price Auctions with Varying Number of Bidders. *Economic Journal*, 103, 868-79.

[30] Laffont, J.J., H. Ossard and Q. Vuong (1995): "Econometrics of First-Price Auctions," *Econometrica*, 63, 953-980.

[31] Lind, B. and C.R. Plott (1991): "The Winner's Curse: Experiments with Buyers and with Sellers, *American Economic Review*, 81, 335-46.

[32] Levin, D. and J.L. Smith (1991): "Some Evidence on the Winner's Curse: A Comment, *American Economic Review*, 81(1), 370-5.

[33] McAfee, R.P. and J. McMillan (1986): "Bidding for Contracts: A Principle-Agent Analysis," *Rand Journal of Economics*, 17, 326-38.

[34] McAfee, R.P. and J. McMillan (1987): "Auctions and Biddings," *Journal of Economic Literature*, 25, 708-747.

[35] McAfee, R.P. and J. McMillan (1987): "Competion for Agency Contracts," *Rand Journal of Economics*, 18, 296-307.

[36] McAfee, R.P. and D. Vincent (1991): "The Afternoon Effect," *CMSEMS Discussion Paper 961.*

[37] McAfee, R.P. and D. Vincent (1992): "Updateing the Reserve Price in Common-Value Auctions," *American Economic Review (Papers and Proceedings)*, 82(2), 512-8.

[38] McFadden, D. (1989): "A Method of Simulated Moments for Estimation of Discrete Response Models," *Econometrica*, 57, 995-1026.

[39] McMillan, J. (1989): "Selling Spectrum Rights," *Journal of Economic Perspectives*, 8(3), 145-62.

[40] Mead, W. (1967): "Natural Resource Disposel Policy - Oral Versus Sealed Bids," *Natural Resource Planning Journal*, M, 195-224 (quoted in [43]).

[41] Milgrom, P.R. (1979): "A Convergence Theorem for Competitive Bidding with Differential Information," *Econometrica*, 49, 679-688.

[42] Milgrom, P. (1981): "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, 49, 921-943.

[43] Milgrom, P. (1987): "Auction Theory," in *Advances in Economic Theory*, edited by T.Bewley. Cambridge: Cambridge University Press.

[44] Milgrom, P.R. and R.J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089-1122.

[45] Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operation Research*, 6, 58-73.

[46] Myerson, R. : "The Basic Theory of Optimal Auctions," *Mimeo. Lecture Notes of J. Swinkels at IAS, Vienna, 1995.*

[47] Paarsch, H.J. (1991): "Empirical Models of Auctions and an Application to British Columbian Timber Sales," *Discussion Paper 91-19, University of British Columbia*, (quoted in: Hendricks and Porter [21]).

[48] Paarsch, H.J. (1992): "Deciding Between the Common and Private Value Paradigms in Empirical Models of Auctions," *Journal of Econometrics*, 51, 192-215.

[49] Pakes, A. and D. Pollard, (1989): "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 57, 1027-1057.

[50] Pesendorfer, M. (1996): "A Study of Collusion in First- Price Auctions," *Mimeo. Yale University.*

[51] Porter, R.H. (1995): The Role of Information in US Offshore and Gas Lease Auctions," *Econometrica*, 63(1), 1-27.

[52] Riley, J. (1988): "Ex Post Information in Auctions," *Review of Economic Studies*, 55, 409-429.

[53] Riley, J. and W. Samuelson (1981): "Optimal Auctions," *American Economic Review*, 71, 381-392.

[54] Samuelson, W. (1986): "Bidding for Contracts," *Management Science*, 32, 1533-1550.

[55] Thiel S.E. (1988): "Some Evidence On The Winner's Curse," *American Economic Review*, 78, 381-95.

[56] Vickrey, W. (1961): "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, 16, 8-37.

[57] Wilson, R. (1977): "A Bidding Model of Perfect Competion," *Review of Economics Studies*, 4, 511-518.

[58] Wilson, R. (1992): "Strategic Analysis of Auctions", in *Handbook of Game Theory with Economic Applications*, edited by R. Aumann and S. Hart. New York: North Holland.

A Appendix

A.1 Definitions

Def.1: The density g has the monotone likelihood ratio property if for all $s' > s$ and $x' > x$, $g(x \mid s)/g(x \mid s') \geq g(x' \mid s)/g(x' \mid s')$. This is equivalent to the affiliation inequality.

A.2 Used Theorems

We now list all theorems used in the paper. Theorem (1)-(5) are concerned with affiliation. They are taken from Milgrom and Weber [44], where the proofs are given, as well. For more detailed definitions the reader is refered to the appendix of that paper.

Theorem 1: Let $f : R^k \to R$. (i) If f is strictly positive and twice continuously differentiable, then f is affiliated if and only if for $i \neq j$, $\partial^2 ln f/\partial x_i \partial x_j \geq 0$. (ii) If $f(x) = g(x)h(x)$ where g and h are nonnegative and affiliated, then f is affiliated.

Theorem 2: If f is affiliated and symmetric in $X_2, \ldots, X_n$, then $S_1, \ldots, S_m, X_1, Y_1, \ldots, Y_{n-1}$ are affiliated.

Theorem 3: If $Z_1, \ldots, Z_k$ are affiliated and $g_1, \ldots, g_k$ are all nondecreasing functions (or all nonincreasing functions), then $g_1(Z_1), \ldots, g_k(Z_k)$ are affiliated.

Theorem 4: If $Z_1, \ldots, Z_k$ are affiliated, then $Z_1, \ldots, Z_{k-1}$ are affiliated.

Theorem 5: Let $Z_1, \ldots, Z_k$ be affiliated and let H be any nondecreasing function. Then the function h defined by

$$\begin{aligned} & h(a_1, b_1; \ldots; a_k, b_k) \\ & \quad = E[H(Z_1, \ldots, Z_k) \mid a_1 \leq Z_1 \leq b_1, \ldots, a_k \leq Z_k \leq b_k] \end{aligned}$$

is nondecreasing in all its arguments. In particular, the functions

$$h(z_1, \ldots, z_l) = E[H(Z_1, \ldots, Z_k) \mid (z_1, \ldots, z_l)]$$

for $l = 1, \ldots, k$ are all nondecreasing.

Institut für Höhere Studien
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna
Austria

Phone: +43-1-599 91-145
Fax: +43-1-599 91-163